

Möhle, Martin (2014) On hitting probabilities of beta coalescents and absorption times of coalescents that come down from infinity. *ALEA Lat. Am. J. Probab. Math. Stat.* **11**, 141–159.

This file contains corrections concerning the article mentioned above. Moreover, two additional remarks are provided, which could be helpful for the reader.

Corrections.

1. Page 153, Case 3: The part '(see remark after Lemma 4.4)' should be removed, since there is no such remark. The statement that there exists a constant C such that $|b_n(k) - 1| \leq Ck/n$ is not correct, since, for $k = n - 1$,

$$b_n(n-1) = \frac{\Gamma(n)\Gamma(\alpha)}{\Gamma(n+\alpha-1)} \sim \Gamma(\alpha)n^{1-\alpha} \rightarrow \infty, \quad n \rightarrow \infty.$$

Therefore, the proof concerning Case 3 has to be modified from page 153, line 12 on as follows. Split the sum $S := \sum_{k=m-1}^{n-1} b_n(k)a_{k-m+1}/k$ into two parts $S = S_1 + S_2$, where

$$S_1 := \sum_{k=m-1}^{\lfloor n/2 \rfloor} b_n(k) \frac{a_{k-m+1}}{k} \quad \text{and} \quad S_2 := \sum_{k=\lfloor n/2 \rfloor + 1}^{n-1} b_n(k) \frac{a_{k-m+1}}{k}.$$

Since $\alpha \in (0, 1)$ it follows that $b_n(k) = \prod_{j=1}^k (n-j)/(n-j+\alpha-1)$ is non-decreasing in $k \in \{1, \dots, n-1\}$ and, hence,

$$|S_2| \leq \frac{b_n(n-1)}{n/2} \sum_{k=m}^{\infty} |a_{k-m+1}| = 2\alpha \frac{b_n(n-1)}{n} \sim 2\alpha \frac{\Gamma(\alpha)}{n^\alpha} \rightarrow 0, \quad n \rightarrow \infty.$$

Therefore, $S_2 \rightarrow 0$ as $n \rightarrow \infty$. It remains to consider S_1 . For $k \leq \lfloor n/2 \rfloor$ we have

$$1 \leq b_n(k) \leq b_n(\lfloor n/2 \rfloor) = \frac{\Gamma(n)\Gamma(n - \lfloor n/2 \rfloor + \alpha - 1)}{\Gamma(n - \lfloor n/2 \rfloor)\Gamma(n + \alpha - 1)} \sim \frac{(n - \lfloor n/2 \rfloor)^{\alpha-1}}{n^{\alpha-1}} \sim 2^{1-\alpha}$$

as $n \rightarrow \infty$. Thus it is allowed to apply dominated convergence to the sum S_1 (interpreted as an integral with respect to the counting measure on $\{m-1, m, \dots\}$), which yields

$$S_1 = \sum_{k=m-1}^{\lfloor n/2 \rfloor} b_n(k) \frac{a_{k-m+1}}{k} \rightarrow \sum_{k=m-1}^{\infty} \frac{a_{k-m+1}}{k} = \int_0^1 \frac{t^{m-1}}{L_\alpha(t)} dt, \quad n \rightarrow \infty.$$

2. Page 157: In the last displayed line $E(\tau)$ should be replaced by $\mathbb{E}(\tau)$.

Remark 1. Theorem 2.1 on p. 143 provides the main formula (2.2) for the hitting probability $h(n, m)$ of the block counting process of the $\beta(2 - \alpha, \alpha)$ -coalescent with parameter $\alpha \in (0, 2)$. We verify below that (2.2) reduces for $\alpha = 1$ (Bolthausen–Sznitman coalescent) to the formula (11) in [1].

Proof. Clearly, for $\alpha = 1$, (2.2) reduces to

$$h(n, m) = (m-1) \sum_{k=m-1}^{n-1} [z^k] \int_0^z \frac{t^{m-1}}{L_1(t)} dt.$$

By (2.3) and (4.11),

$$[z^k] \int_0^z \frac{t^{m-1}}{L_1(t)} dt = \frac{1}{k} \sum_{j=1}^{k-m+1} (-1)^j \sum_{\substack{n_1, \dots, n_j \in \mathbb{N} \\ n_1 + \dots + n_j = k-m+1}} \frac{1}{(n_1+1) \cdots (n_j+1)} = \frac{a_{k-m+1}}{k},$$

where a_0, a_1, \dots are the coefficients in the Taylor expansion $z/L_1(z) = \sum_{j=0}^{\infty} a_j z^j$. Thus,

$$h(n, m) = (m-1) \sum_{k=m-1}^{n-1} \frac{a_{k-m+1}}{k} = (m-1) \sum_{j=0}^{n-m} \frac{a_j}{j+m-1},$$

which is Eq. (11) of [1]. Note that in [1] the alternative representation $a_j = (-1)^j / j! \int_0^1 (x)_j dx$ for the coefficient a_j is used (see [1, Lemma 3.1]).

Remark 2. On p. 155 at the beginning of the proof of Theorem 3.3 it is stated that τ_{∞} almost surely coincides with τ . We verify below the slightly stronger result that $\tau_{\infty}(\omega) = \tau(\omega)$ for all $\omega \in \Omega$.

Proof. Fix $\omega \in \Omega$. Clearly, $\{t > 0 : N_t(\omega) = 1\} \subseteq \{t > 0 : N_t^{(n)}(\omega) = 1\}$ for all $n \in \mathbb{N}$ and, hence, $\tau_n(\omega) := \inf\{t > 0 : N_t^{(n)}(\omega) = 1\} \leq \inf\{t > 0 : N_t(\omega) = 1\} =: \tau$ for all $n \in \mathbb{N}$. Taking the limit $n \rightarrow \infty$ it follows that $\tau_{\infty}(\omega) \leq \tau(\omega)$.

Assume now that $\tau_{\infty}(\omega) < \tau(\omega)$. Then there exists $t = t(\omega) \in (0, \infty)$ such that $\tau_{\infty}(\omega) < t < \tau(\omega)$. Since $\tau_n(\omega) \leq \tau_{\infty}(\omega)$ it follows that $\tau_n(\omega) < t$ and hence $N_t^{(n)}(\omega) = 1$ for all $n \in \mathbb{N}$. But this implies that $N_t(\omega) = 1$ and hence $\tau(\omega) \leq t$ in contradiction to $t < \tau(\omega)$. Thus, the assumption is wrong and we have $\tau_{\infty}(\omega) = \tau(\omega)$.

References

- [1] MÖHLE, M. (2014) Asymptotic hitting probabilities for the Bolthausen–Sznitman coalescent. *J. Appl. Probab.* **51A**, 87-97. MR3317352